

# The lower limits of charge and radiated energy of antennas as predicted by classical electrodynamics – Revisited

Vernon Cooray<sup>1</sup> and Gerald Cooray<sup>2</sup>

<sup>1</sup>Department of engineering sciences, Uppsala University, Sweden

<sup>2</sup>Karolinska Institute, Stockholm, Sweden

**Abstract**— Power radiated by an oscillating current in an antenna occurs in bursts of duration  $T/2$ , where  $T$  is the period of oscillation. The results obtained here, based purely on classical electrodynamics, can be summarized by the inequality  $U \geq h\nu \rightarrow q_0 \geq e$  where  $U$  is the energy radiated in a single power burst of duration  $T/2$ ,  $h$  is the Planck constant,  $\nu$  is the frequency of oscillation and  $q_0$  is the magnitude of the oscillating charge associated with the oscillating current. The condition  $U = h\nu \rightarrow q_0 = e$  is obtained when the length of the antenna is stretched and its radius is compressed to the limiting values allowed by nature.

Keywords—Elementary charge, Bohr radius, Hubble radius, Energy density of vacuum.

## I. INTRODUCTION

The power dissipated by an antenna excited by an oscillating current takes place in bursts of duration  $T/2$ , where  $T$  is the period of oscillation. This paper is concerned with the energy dissipated within such a single burst. The results presented here extend and improve part of the research work carried out in [1]. The details of the analysis given here were published in a recent paper [2].

An antenna of length  $L/2$  excited by an oscillating current is located over a perfectly conducting ground plane. The energy radiated over any given time interval by this antenna oscillates as a function of  $L/\lambda$  [3]. For very large values of  $L/\lambda$ , only an infinitesimal fractional change of this ratio is needed to push the energy from a maximum to the next minimum. Thus, for very large values of  $L/\lambda$ , the median value of this oscillating energy is considered.

## II. RESULTS

Under ideal conditions when all the losses associated with the current propagation along the antenna can be neglected, the median value of the energy radiated within a single burst of duration  $T/2$  is given by [3]

$$U_{med} = \frac{q_0^2 \pi \nu}{4 \epsilon_0 c} \{ \gamma + \ln(2\pi L / \lambda) \} \quad (1)$$

In the above equation  $\gamma$  is the Euler's constant,  $q_0$  is the

magnitude of the oscillating charge,  $\lambda$  is the wavelength and  $\nu$  is the frequency of oscillation. For a given charge  $q_0$ , the median energy increases with increasing length and decreasing wavelength. The smallest value of the wavelength that one can plug into the equation is in the order of the antenna radius. The smallest possible radius of an antenna that can exist in nature is equal to the Bohr radius,  $a_0$ . The largest possible value of the antenna length that one can have in nature is equal to the ultimate size of the universe and this is given by the final (or the steady) value of Hubble radius,  $R_\infty$ . Thus, the upper limit of the median energy dissipated within a single power burst of duration  $T/2$  for a given charge is given by,

$$U_{max} = \frac{q_0^2 \pi \nu}{4 \epsilon_0 c} \{ \gamma + \ln(2\pi R_\infty / a_0) \} \quad (2)$$

Equation (2) predicts that when  $q_0 = e$ , where  $e$  is the elementary charge,  $U_{max} \approx h\nu$  where  $h$  is the Planck constant. Since  $U_{max}$  is the maximum energy that can be radiated within a single burst of power for any given charge, the results can be summarized by the inequality  $U \geq h\nu \rightarrow q_0 \geq e$  where  $U$  is the energy radiated in a single power burst of duration  $T/2$ ,  $h$  is the Planck constant,  $\nu$  is the frequency of oscillation and  $q_0$  is the magnitude of the oscillating charge associated with the current. Since  $R_\infty = c^2 \sqrt{3/8\pi G \rho_\Lambda}$ , where  $G$  is the gravitational constant and  $\rho_\Lambda$  is the vacuum energy density, replacing  $U_{max}$  by  $h\nu$  and  $q_0$  by  $e$  in Equation (2) will generate an expression for the vacuum energy density in terms of other known fundamental constants.

## REFERENCES

- [1] Cooray, V. and G. Cooray, (2016), On the remarkable features of the lower limits of charge and the radiated energy of antennas as predicted by classical electrodynamics. *Atmosphere*, 7, 64.
- [2] Cooray, V. and G. Cooray, Remarkable predictions of classical electrodynamics on elementary charge and the energy density of vacuum, *Journal of Electromagnetic Analysis and Application*, vol. 10, no. 5, 2018.
- [3] Balanis, C.A. *Antenna Theory: Analysis and Design*; Harper and Row Publishers: New York, NY, USA, 1982.